Explanatory document related to the paper entitled "Signal Processing Applied to Chemically Inspired Communication Protocols"

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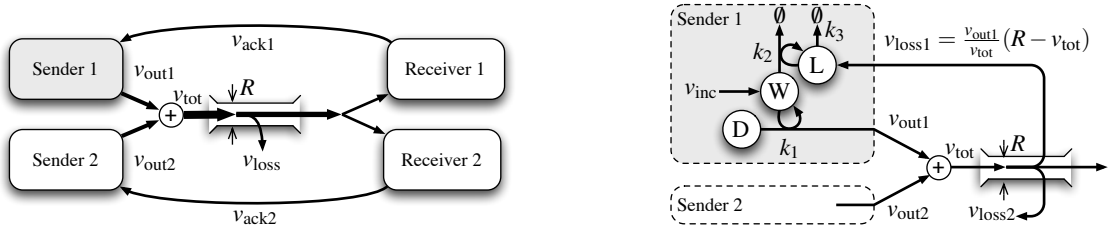
Abstract

This document provides an auxiliary formal material to assist the understanding of the analysis process and equations presented in sections III and IV of the paper "Signal Processing Applied to Chemically Inspired Communication Protocols" (IEEE ICC 2012 conference, Ottawa, Canada).

MATHEMATICAL DETAILS REGARDING THE DERIVATION OF THE TRANSFER FUNCTION (TF) OF THE CHEMICAL CONGESTION CONTROL ALGORITHM (C_3A).



(a) Channel sharing communication model

(b) The chemical reaction network of C_3A

Fig. 1: A general model of two packet streams sharing the same channel (a), and the chemical reaction network of C_3A that implements the chemical congestion control algorithm in sender 1 (b).

The dynamics of C_3A in the scenario of Fig. 1(b) (section II-B of the paper) is described by the following system of coupled non-linear ODEs:

$$\dot{c}_W = \overbrace{v_{inc}}^{\text{lin. increase}} - \overbrace{k_2 c_W c_L}^{\text{exp. decrease}}$$

$$\dot{c}_L = \underbrace{(k_1 c_W c_D + v_{out2} - R)\frac{k_1 c_W c_D}{k_1 c_W c_D + v_{out2}}}_{\textit{pro rata}\text{ packet loss}} - \underbrace{k_3 c_L}_{\text{decay of L}}$$

The first step towards the calculation of the *Transfer Function* (TF) of the linearized version of the system in Fig. 1(b) is to find the steady states by solving $\dot{\mathbf{c}}(t) = \mathbf{\Psi} \cdot \mathbf{v}\left(\mathbf{c}(t), \mathbf{p}(t)\right) = 0$, where the stoichiometric matrix is

$$\mathbf{\Psi} = \begin{bmatrix} 1 & 0 & -1 & 0 \\ 0 & 1 & 0 & -1 \end{bmatrix}$$

the reaction vector is

$$\mathbf{v} = \begin{bmatrix} v_{inc} & k_1 c_W \left(1 - \frac{R}{k_1 c_W + v_{out2}}\right) & k_2 c_W c_L & k_3 c_L \end{bmatrix}^T \tag{2}$$

the state vector is

$$\mathbf{c} = \begin{bmatrix} c_W & c_L \end{bmatrix}^T \tag{3}$$

and the perturbation vector is

$$\mathbf{p} = \begin{bmatrix} v_{\text{out2}} \end{bmatrix}$$

In (2) and (3), we have omitted the concentration of D-species, c_{D}. Indeed, as we have pointed out in sec.II-B of the paper, the data species D is buffered and contains the constant amount of *one* instance: $c_{\text{D}} = 1\,\text{pkt} = \text{const.}$. The resulting solution set $\mathbf{c}^* = \begin{bmatrix} c_{\text{W}}^* & c_{\text{L}}^* \end{bmatrix}$ of system steady-states consists of the following two terms:

$$c_{\text{W}}^* = \begin{bmatrix} \frac{R+\sqrt{\left(\frac{k_2R^2+4k_1k_3v_{\text{inc}}}{k_2}\right)}}{2k_1} & \frac{R-\sqrt{\left(\frac{k_2R^2+4k_1k_3v_{\text{inc}}}{k_2}\right)}}{2k_1} \end{bmatrix}$$

$$c_{\text{L}}^* = \begin{bmatrix} -\frac{\sqrt{\left(\frac{k_2R^2+4k_1k_3v_{\text{inc}}}{k_2}\right)}+R}{2k_3} & \frac{\sqrt{\left(\frac{k_2R^2+4k_1k_3v_{\text{inc}}}{k_2}\right)}-R}{2k_3} \end{bmatrix}$$

Note that since system steady-states represent molecular concentrations, we must take into account positive values only, i.e. the first term of c_{W}^*-vector and the second term of c_{L}^*-vector.

According to sec.III-B of the paper, we want to represent the system as a Linear Time Invariant (LTI) system:

$$\dot{\mathbf{x}}(t) = \mathbf{A} \cdot \mathbf{x}(t) + \mathbf{B} \cdot \mathbf{u}(t) \tag{4a}$$

$$\mathbf{y}(t) = \mathbf{C} \cdot \mathbf{x}(t) + \mathbf{D} \cdot \mathbf{u}(t) \tag{4b}$$

where $\mathbf{x}(t) = \mathbf{c}(t) - \mathbf{c}^*$ and $\mathbf{u}(t) = \mathbf{p}(t) - \mathbf{p}^*$ denote deviations from nominal states and input signal values, respectively. The term $\mathbf{y}$ represents the output vector of the quantities of our interest. The nature of $\mathbf{y}$-elements is defined by the choice of $\mathbf{C}$ and $\mathbf{D}$ matrix. In our case, since we set $\mathbf{C} = \mathbf{I}$ and $\mathbf{D} = \mathbf{0}$, $\mathbf{y}$-elements represent the variation of concentrations of W-species and L-species. The state matrix $\mathbf{A}$ and the input matrix $\mathbf{B}$ are evaluated around the fixed point $(\mathbf{c}^*, \mathbf{p}^*)$ such that:

$$\mathbf{A} = \mathbf{\Psi} \cdot \left.\frac{\partial \mathbf{v}}{\partial \mathbf{c}}\right|_{(\mathbf{c}^*, \mathbf{p}^*)} \qquad \mathbf{B} = \mathbf{\Psi} \cdot \left.\frac{\partial \mathbf{v}}{\partial \mathbf{p}}\right|_{(\mathbf{c}^*, \mathbf{p}^*)}$$

According to the argumentation in sec.III-C of the paper, we can calculate the 2-element TF matrix that reports the transient behavior of W-molecule and L-molecule concentrations w.r.t. perturbations on the transmission rate of sender 2:

$$\mathbf{H}(s) = \begin{bmatrix} C_{\text{W}}(s) & C_{\text{L}}(s) \end{bmatrix} = \mathbf{C}\left(s \cdot \mathbf{I} - \mathbf{A}\right)^{-1}\mathbf{B} + \mathbf{D}$$

Since we look at the behavior of sender 1 (namely, we look at sender 1's transmission rate adaptation) we focus on the element which describes the transient behavior of W-molecule concentration (this term affects proportionally the transmission rate of sender 1):

$$\mathbf{H}_{C_3A}(s) = k_1 C_{\text{W}}(s) = -\frac{Rk_2k_3}{s^2k_3 + s\left(\frac{2k_3^2+k_2\alpha-k_2R}{2}\right) + k_2k_3\alpha} \tag{5}$$

where $\alpha = \sqrt{\frac{k_2R^2+4k_1k_3v_{\text{inc}}}{k_2}}$. Assuming sufficiently large link capacity values, i.e. $R \gg \sqrt{\frac{4k_1k_3v_{\text{inc}}}{k_2}}$, and recalling that all involved variables in (5) represent positive quantities, the TF can be simplified to:

$$H_{C_3A}(s) = -\frac{Rk_2}{s^2 + sk_3 + Rk_2}$$

□